A New Structural Analysis/Synthesis Capability—ACCESS 1

L. A. Schmit* and H. Miura†
University of California, Los Angeles, Los Angeles, Calif.

The creation of an efficient automated capability for minimum weight design of structures is reported. The ACCESS 1 (Approximations Concept Code for Efficient Structural-Synthesis) computer program combines finite-element analysis techniques and mathematical programing algorithms using an innovative collection of approximation concepts. Design variable linking, constraint deletion techniques, and approximate analysis methods are used to generate a sequence of small explicit mathematical programing problems which retain the essential features of the design problem. Organization of the finite-element analysis is carefully matched to the design optimization task. The efficiency of the ACCESS 1 program is demonstrated by giving results for several example problems.

Nomenclature

A_i	= cross-sectional area of ith truss element
A	= vector representation of a set of $A_i, i = 1,2,\ldots,I_T$
a	= sequence number of unconstrained minimization for the interior penalty function formulation
a'	= number of unconstrained minimizations to be carried out in one stage of design procedure
B	= total number of independent design variables after linking
$b(i)$	= independent design variable number associated with the ith finite element
$[\tilde{C}]$	= partial inverse of the system stiffness matrix $[K]$
C_b	= constant in the weight equation [cf. Eq. (13)]
c_a	= reduction ratio of the penalty multiplier r_a
c_i	= weight of the ith finite element for $D_i = 1.0$
D_i	= sizing variable for the ith finite element
$D_i^{(L)}, D_i^{(U)}$	= lower and upper limits of the ith sizing variable
D	= vector representation of a set of D_i, $i = 1,2,\ldots,I$
$[\mathfrak{D}]$	= diagonal matrix
d	= move distance variable in the direction S_m
E	= modulus of elasticity
e_j	= vector with J elements all of which are zero except for the jth element which has unit value
$\tilde{H}_q^{(p)}(\alpha)$	= extended interior penalty function defined by Eq. (33)
$h_q(\alpha)$	= qth constraint function
$\tilde{h}_q^{(p)}(\alpha)$	= explicit approximation of $h_q(\alpha)$ for the pth stage
I	= total number of finite elements in the analysis model
I_T, I_C, I_S	= total number of finite elements in TRUSS, CST, and SSP categories
i	= index for finite elements
J	= number of unknown displacement degrees of freedom
J'	= set of integers identifying the displacement degrees of freedom defining the values of the retained constraints
J_u	= set of integers identifying the constrained displacement degrees of freedom
j	= index for the independent displacement degrees of freedom in the finite-element model
$[K]$	= system stiffness matrix
K	= total number of load conditions
K'	= set of load conditions contributing at least one behavioral constraint after deletion
$[\tilde{k}_{\ell(i)}]$	= element unit stiffness matrix of the ℓth configuration group in the local coordinate system
k	= index for independent static load conditions
$[\mathfrak{L}]$	= decomposed lower triangular matrix
$\ell(i)$	= configuration group number associated with the ith finite element
$M(D)$	= objective function in terms of D
m	= index for one-dimensional minimization step
P_k	= load vector for the kth load condition
p	= index for the stages in the iterative design procedure
Q	= total number of constraints
$Q_R^{(p)}$	= number of constraints retained during the pth stage of design process
q	= index for the constraint functions
r_a	= penalty multiplier in the interior penalty function formulation [see Eq. (32)]
S_m	= normalized direction vector
s_m	= direction vector
$T_{ib(i)}$	= linking table entry, defined by Eq. (8)
TBV	= truncation boundary value
TV_{bk}	= total number of retained V_{bk} vectors
t_i	= thickness of the ith CST element
t	= vector representation of a set of t_i: $i = 1,2,\ldots,I_c$
u_{jk}	= jth displacement degree of freedom for the kth load condition
$u_j^{(L)}, u_j^{(U)}$	= lower and upper limits on jth displacement degree of freedom

Presented as Paper 75-763 at the AIAA/ASME/SAE 16th Structures, Structural Dynamics and Materials Conference, Denver, Colo., May 27-29, 1975; submitted June 5, 1975; revision received October 1, 1975. This research was supported by NASA Research Grant NGR 05-007-337.

Index categories: Structural Design, Optimal; Structural Static Analysis.

*Professor of Engineering and Applied Science. Associate Fellow AIAA.

†Assistant Professor in Residence and Research Engineer. Member AIAA.

$\boldsymbol{u}_k$ = vector representation of u_{jk}, $j=1,2,\ldots,J$
$\tilde{u}_{jk}(\boldsymbol{\alpha})$ = explicit approximation of u_{jk}
$\boldsymbol{V}_{bk}$ = pseudo load vector defined by Eq. (23)
$W(\boldsymbol{\alpha})$ = objective function in terms of $\boldsymbol{\alpha}$
$\alpha_{b(i)}, \alpha_b$ = reciprocal of the independent design variable δ_b after linking
$\boldsymbol{\alpha}$ = vector representation of a set of α_b, $b=1,\ldots,B$
β_i = reciprocal of the sizing variable for the ith element
$\delta_{b(i)}$ = independent design variable after linking
ϵ = transition parameter in extended penalty function definition
ϵ' = a small constant used in Eq. (25)
$[\Lambda_i]$ = coordinate transformation matrix for the ith finite element
ν = Poisson's ratio
ρ = specific weight
σ_{ik} = axial stress in truss element i under load condition k
$\sigma_{xik}, \sigma_{yik}$ = normal stress components in the ith CST or SSP element, under load condition k
σ_{ai} = allowable equivalent stress
σ_{eik} = Von Misses equivalent stress in element i under load condition k, defined by Eq. (1)
$\sigma_i^{(L)}, \sigma_i^{(U)}$ = lower and upper limits of stress for the ith element in each type of finite element
τ_i = thickness of the ith SSP element
$\boldsymbol{\tau}$ = vector representation of the set of τ_i, $i=1,2,\ldots,I_s$
τ_{xyik} = shear stress in the ith CST or SSP element under load condition k
$\phi_e^{(p)}(\boldsymbol{\alpha}, r_a)$ = sum of the objective function and the extended penalty functions, defined by Eq. (32).

I. Introduction

IN 1960 it was first suggested[1] that finite-element structural analysis methods and nonlinear programing techniques could be coupled together to generate automated structural design optimization capabilities. The decade 1960-1970 saw the emergence of two major finite-element based analysis/synthesis capabilities[2-4] using mathematical programing techniques. During the period from 1968-1971 there was a growing awareness that these first generation programs required far too many analyses and inordinately long computer run times to solve design optimization problems of only modest proportions. Many investigators,[5-13] believing that an insurmountable efficiency barrier had been encountered, turned away from the mathematical programing approach and focused renewed efforts on the implementation of recursive redesign procedures based on fully stressed design concepts and discretized optimality criteria methods. This paper reports some recent results of an ongoing research effort guided by the conviction that the innovative use of approximation concepts make it possible to create efficient structural analysis/synthesis capabilities based on combining finite-element structural analysis methods and nonlinear programing techniques.

The use of approximation concepts was previously explored in the concept of truss structures,[14] subject to static stress and displacement constraints for a multiplicity of distinct loading conditions as well as minimum member size limitations. The mathematical programing algorithm used for the earlier truss studies was an adaptation of the method of inscribed hyperspheres.[15] The research results set forth in this paper go well beyond those previously reported in several important respects, namely: 1) Structures idealized by truss, constant strain triangle, and symmetric shear panel finite elements can now be treated routinely. 2) Special attention has been focused on organizing the finite-element structural analysis so as to facilitate efficient analysis of several alternative designs having the same basic configuration but different proportions. 3) A selective sensitivity analysis has been implemented in which only those partial derivatives actually needed to construct explicit approximations for critical and near critical constraints are computed. 4) The design procedure has been broken down into two major modular tasks, specifically approximate problem generation and application of a mathematical programing algorithm to produce an improved design at each stage in the iterative design procedure. 5) The limitations inherent to the use of the method of inscribed hyperspheres[15] have been removed by selecting alternative optimization algorithms that do not necessarily require linearization of the inequality constraint functions and the objective function.

These advances are embodied in the ACCESS 1 computer program. The technical foundations of this program are described in the sequal. ACCESS 1 is a new type of structural analysis/synthesis capability which achieves excellent efficiency through the coordinated implementation of several approximation concepts.

II. Scope and Primary Formulation

The scope of ACCESS 1 is such that it embraces a significant class of structural design optimization problems. It is assumed that the topological form, the geometric configuration, and the structural materials to be employed are preassigned parameters. Attention is focused on two- and three-dimensional structural systems that can be idealized using three types of finite elements, namely truss elements of uniform cross-sectional area (TRUSS), isotropic constant strain triangular membrance elements of uniform thickness (CST), and isotropic symmetric shear panel elements of uniform thickness (SSP). It should be noted that the SSP element is particularly useful for modeling midsurface symmetric thin wing structures because it permits uncoupling inplane displacement response from bending and twisting behavior. The basic assumptions and the resulting local stiffness matrices for the TRUSS, CST, and SSP elements used in ACCESS 1 are given in Appendix A of Ref. 16. The physically significant design variables are understood to be the cross-sectional areas of the TRUSS elements and the thickness of the CST and SSP elements. Multiple static loading conditions are considered and they are assumed to be independent of the design variables.

The ACCESS 1 program considers stress and displacement constraints under each of several distinct loading conditions. Separate upper and lower limits may be placed on each independent displacement component (expressed in the reference coordinate system). The limits on each displacement component are assumed to be the same for all loading conditions. Independent upper and lower stress limitations may be specified for TRUSS elements. For CST and SSP elements an upper limit (σ_{ai}) is placed on the Von Mises equivalent stress (σ_{eik}) that is

$$\sigma_{eik} = (\sigma_{xik}^2 + \sigma_{yik}^2 - \sigma_{xik}\sigma_{yik} + 3\tau_{xyik}^2)^{1/2} \leq \sigma_{ai} \tag{1}$$

where the subscript i refers to the element and the subscript k refers to the load condition. The limiting stress values σ_{ai} are assumed to be independent of the loading condition. In addition to the foregoing behavioral constraints, minimum and maximum member sizes can be specified for each finite element. The objective function is taken to be the total weight of the finite-element idealization.

The primary formulation of the structural synthesis problem dealt with by ACCESS 1 can be stated as follows:

Given the preassigned parameters and the load condition;

Find the vector of design variables

$$\boldsymbol{D}^T = [\boldsymbol{A}^T, \boldsymbol{t}^T, \boldsymbol{\tau}^T] \tag{2}$$

such that the displacement constraints

$$u_j^{(L)} \leq u_{jk}(\boldsymbol{D}) \leq u_j^{(U)};\ j \in J_u;\quad k=1,2,\ldots,K \tag{3}$$

the stress constraints

$$\sigma_i^{(L)} \leq \sigma_{ik}(\boldsymbol{D}) \leq \sigma_i^{(U)};\ i \in I_T;\quad k=1,2,\ldots,K \tag{4}$$

$$\sigma_{eik}(\boldsymbol{D}) \leq \sigma_{ai};\ i \in I_C \text{ and } I_S;\quad k=1,2,\ldots,K \tag{5}$$

and the member size constraints

$$D_i^{(L)} \leq D_i \leq D_i^{(U)}\ ;\qquad i=1,2,\ldots,I \tag{6}$$

are satisfied while the objective function

$$M(\boldsymbol{D}) = \sum_{i=1}^{I} c_i D_i \tag{7}$$

is minimized. This primary formulation involves a large number of design variables and an even larger number of inequality constraints. Furthermore, the displacement and stress constraints embodied in Eqs. (3-5) are implicit functions of the design variables whose evaluation requires execution of a conventional finite-element structural analysis.

III. Approximation Concepts

The ACCESS 1 efficiency is achieved without loss of generality. This is accomplished by replacing the primary formualtion [Eqs. (2-7)] with a sequence of easier substitute problems, while preserving the essential characteristics of the original problem. Generation of the substitute problems involves the coordinated use of the following approximation concepts: 1) reduction of the number of independent design variables by design variable linking; 2) reduction of the number of constraints considered at each stage in the design process by temporary deletion of inactive and redundant constraints; 3) the construction of high quality explicit approximations for retained constraint functions.

Through the insightful use of these approximation concepts the mathematical programing problem faced at each stage in the design procedure is rendered small and explicit while retaining the essential features of the primary design optimization problem.

Design Variable Linking

In many structural design optimization problems the number of finite elements needed in the analysis to adequately predict the behavior is much larger than the number of design variables required to properly describe the design problem at hand. In general, the idealization and discretization decisions leading to the analysis model should be independent of the judgments leading to the design model. Frequently, it is neither necessary nor desirable for each finite element in the structural analysis model to have its own independent design variable.

Design variable linking simply fixes the relative size of some preselected group of finite elements, so that one independent design variable controls the size of all finite elements in that linking group. Design variable linking is accomplished by relating the ith component of the vector of design variables (D_i) to the bth independent design variable after linking (δ_b) according to the prescription

$$D_i = T_{ib(i)}\,\delta_{b(i)} \tag{8}$$

where the $T_{ib(i)}$ are taken to be the value of D_i at the beginning of a particular stage in the design procedure, and it is understood that the initial values of he $\delta_{b(i)}$ for each stage are all plus one. Note that the subscript $b(i)$ denotes an integer element of a pointer vector $\boldsymbol{b}$ which given (i) identifies the independent variable b to which the size of the finite element i is linked. In ACCESS 1 design variable linking is limited to prespecified groups of finite elements that are all of the same type (i.e., all TRUSS, all CST or all SSP elements).

For the class of problems treated by ACCESS 1 it is advantageous to use reciprocal design variables β_i defined as follows

$$\beta_i = (1/D_i) \tag{9}$$

This change of variables is motivated by the fact that for statically determinate structures idealized by elements in which the stress is inversely proportional to the sizing design variables D_i (e.g., TRUSS, CST and SSP elements), both stress and displacement response are strictly linear in the reciprocal variables β_i. As a consequence, using reciprocal variables generally improves the quality of linear approximations of the stress and displacement response for moderately indeterminate structures. Let $\alpha_{b(i)}$ denote the reciprocal of the bth independent design variable after linking, that is

$$\alpha_{b(i)} = \frac{1}{\delta_{b(i)}} \tag{10}$$

Then from Eqs. (8-10) it follows that

$$\frac{1}{D_i} = \beta_i = \frac{\alpha_{b(i)}}{T_{ib(i)}} \tag{11}$$

Introducing the change of variables represented by Eq. (11), at the beginning of each stage in the design procedure, reduces the number of variables via linking and introduces normalized reciprocal variables. It should be recognized that while the design variable linking is invariant from stage to stage, the linked reciprocal variables $\alpha_{b(i)}$ are normalized to have unit value at the beginning of each stage so as to enhance the numerical stability of the optimization algorithm(s).

Depending upon the underlying motivation, design variable linking may be viewed as an improvement of the original problem statement or as a special type of basis reduction. If design variable linking is used to impose symmetry requirements or to impose fabrication and cost control considerations then it sharpens the problem statement. Under these circumstances design variable linking restricts the search for an optimum design to the subspace in which the desired solution must reside. On the other hand, if design variable linking is based on designer insight, prior experience, or simply arbitrary decisions aimed primarily at reducing the number of independent design variables, then it represents a special type of basis reduction and the designs obtained with ACCESS 1 are at least feasible upper bound approximations of the optimum.

In any event the primary problem formulation [Eqs. (2-7)], when restated in terms of linked reciprocal variables (α_b; $b = 1,2,\ldots,B$), has the general form; find $\boldsymbol{\alpha}$ such that

$$h_q(\boldsymbol{\alpha}) \geq 0;\quad q=1,2,\ldots,Q \tag{12}$$

and

$$W(\boldsymbol{\alpha}) = \sum_{b=1}^{B} C_b/\alpha_b \rightarrow \min \tag{13}$$

where the inequality constraints have been normalized so that typically they take on values between 0 and 1 in the feasible region. Furthermore, it is to be understood that strictly redundant member size constraints have been permanently deleted.

At this juncture the number of independent design variables has been reduced from I to B; however, the number of constraints Q is still very large and most of the $h_q(\boldsymbol{\alpha})$ are still implicit functions of the design variables (α) whose evaluation requires lengthy analysis computations.

Constraint Deletion

The constraint deletion techniques employed in ACCESS 1 are, to a substantial degree, nothing more than the computer implementation of traditional design practice. It is recognized that during each stage of an iterative design process it is only necessary to consider critical and potentially critical constraints. The basic idea then is to temporarily ignore redundant and noncritical constraints that are likely to have little influence on the design process during an upcoming stage.

For each type of constraint (i.e., displacement, stress, or member size) independent *t*runcation *b*oundary *v*alues (TBV's) are used for temporary deletion of constraints that can be ignored during the next stage of the design optimization process. In ACCESS 1 the general form of the deletion criterion for each type of constraint is

$$\text{if } h_q(\alpha) > \text{ TBV then delete} \tag{14}$$

It is to be understood that the TBV's for each type of constraint, are dynamic factors that are decreased prior to generating each new approximate problem statement. Input parameters control the initial values, rates of decrease, and minimum values of the TBV's. In a representative example, the stress constraint TBV could have an initial value of 0.9 which is reduced to a minimum value of 0.4 in about 12 design stages. The use of dynamic TBV's recognizes that the constraints governing the optimum design are not known at the outset, rather their identity emerges gradually during the iterative design process.

In ACCESS 1 a further reduction in the number of constraints retained during a stage is accomplished by deleting essentially redundant stress constraints. Let each group of finite elements for which sizing is controlled by a single independent design variable (after linking), constitute a design variable linking group region. Then for each design variable linking group region (b) for each loading condition (k) only the most critical stress constraint, of those surviving the previously described truncation process [see Eq. (14)], is retained. Therefore the maximum number of stress constraints that may be present during any stage is $B \times K$, however, the number actually retained is usually much smaller.

The use of design variable linking groups to define regions makes shifting of the critical constraints during a design stage rather unlikely. This is because changes in the independent design variables will primarily cause force redistributions between regions rather than within design variable linking group regions. In this connection it may be helpful to recall that, for statically determinate structures, the identity of the most critical stress constraint in each region for each load condition is independent of the set of values assigned to the design variables. Constraints that survive the double deletion process are treated as critical or potentially critical constraints during the upcoming stage of the design process and they are logged into a posture table that is a vector of pointers containing the label numbers of these constraints.

Explicit Approximations

Since the objective function is simple and explicit in the exact form given by Eq. (13), no approximation is required and the first and second partial derivatives of $W(\alpha)$ with respect to the linked reciprocal variables are

$$\frac{\partial W}{\partial \alpha}(\alpha) = -C_b/\alpha_b^2;\; b=1,2,\ldots,B \tag{15}$$

and

$$\frac{\partial^2 W}{\partial \alpha_b \partial \alpha_c}(\alpha) = 2\delta_{bc} C_b/\alpha_b^3;\quad \begin{matrix} b=1,2,\ldots,B \\ c=1,2,\ldots,B \end{matrix} \tag{16}$$

Note that for $\alpha_b \geq \alpha_b^{(L)} > 0$ the first derivatives are all negative and the second derivatives are all positive.

In ACCESS 1 first-order Taylor series expansions are used to construct explicit approximations for stress and displacement reponse quantities in terms of the linked reciprocal design variables (α_b). For idealized structures modeled with TRUSS, CST, and SSP finite elements these first-order Taylor series representations are known to be of high quality for moderately indeterminate structures and they are exact for statically determinate structures.

The first-order Taylor series approximation of the jth displacement degree of freedom under the kth load condition, based upon analysis of the current trial design $\boldsymbol{\alpha}_p$ can be expressed as follows:

$$u_{jk}(\boldsymbol{\alpha}) \approx \tilde{u}_{jk}(\boldsymbol{\alpha}) = u_{jk}(\boldsymbol{\alpha}_p) + (\boldsymbol{\alpha} - \boldsymbol{\alpha}_p)^T \nabla u_{jk}(\boldsymbol{\alpha}_p) \tag{17}$$

where the components of the vector $\nabla u_{jk}(\boldsymbol{\alpha}_p)$ are the partial derivatives $\partial u_{jk}/\partial \alpha_b(\boldsymbol{\alpha}_p)$ for $b = 1,2,\ldots,B$.

For a given design the displacement solutions for each of K loading conditions are obtained by solving the following familiar set of equations,

$$[K]\, \boldsymbol{u}_k = \boldsymbol{P}_k\, ;\; k = 1,2,\ldots,K \tag{18}$$

Since ACCESS 1 is an all in-core computer program the system stiffness matrix assembly scheme employed is designed to minimize storage requirements. This is accomplished by storing only one local (unit) stiffness matrix $[\tilde{k}_{\ell(i)}]$ for each finite-element configuration group. Finite elements of the same type (i.e., all TRUSS, all CST, or all SSP) are said to belong to the same configuration group if they have identical configuration and material properties. The system stiffness matrix $[K]$ may be expressed as follows

$$[K] = \sum_{i=1}^{I} D_i [\Lambda_i]^T [\tilde{k}_{\ell(i)}]\; [\Lambda_i] \tag{19}$$

where the subscript $\ell(i)$ denotes the configuration group ℓ to which element i belongs. To express the system stiffness matrix $[K]$ in terms of the linked reciprocal design variables eliminate D_i from Eq. (19) using Eq. (11) then

$$[K] = \sum_{i=1}^{I} \frac{T_{ib(i)}}{\alpha_{b(i)}} [\Lambda_i]^T [\tilde{k}_{\ell(i)}]\; [\Lambda_i] \tag{20}$$

Returning to Eq. (18) it is noted that the linear equation solver built into ACCESS 1 is based on the algorithm given in Ref. 17. The system stiffness matrix $[K]$ is decomposed into the product of three matrices, namely

$$[K] = [\mathfrak{L}]\; [\mathfrak{D}]\; [\mathfrak{L}]^T \tag{21}$$

where $[\mathfrak{L}]$ is a lower half triangular matrix, and $[\mathfrak{D}]$ is a diagonal matrix. After decomposition a sequence of back and forward substitutions for each $\boldsymbol{P}_k$ leads to the corresponding displacement response vector $\boldsymbol{u}_k$.

The selective sensitivity concept is implemented by the ACCESS 1 program. In other words, the only partial derivatives evaluated at a particular stage, are those needed to construct explicit approximations for constraints surviving the deletion process. The necessary partial derivatives are obtained without recourse to finite difference techniques.

For any particular trial design $\boldsymbol{\alpha}_p$ it is known that the stresses in a finite element can be readily determined if the displacement degrees of freedom $u_{jk}(\boldsymbol{\alpha}_p)$ are known. In ACCESS 1 the stress and displacement constraints retained at a particular stage are scanned and the subset of displacement degrees of freedom $j \epsilon J'$ defining the values of the retained constraints is identified. This scanning also determines which, if any, load conditions do not contribute any stress or displacement constraints to the retained set of constraints. Let

K' denote the set of load conditions that contribute at least one stress or displacement constraint to the set of constraints retained after completing the double deletion process previously described.

Assuming the load vectors P_k are independent of the design variables, implicit differentiation of Eq. (18) with respect to the linked reciprocal design variables α_b yields

$$[K]\frac{\partial u_k}{\partial \alpha_b} = V_{bk};\quad \begin{matrix} k=1,2,...,K \\ b=1,2,...,B \end{matrix} \tag{22}$$

where the vectors

$$V_{bk} = -\left[\frac{\partial K}{\partial \alpha_b}\right] u_k \tag{23}$$

are sometimes called pseudo load vectors. In the ACCESS 1 program the matrices $[\partial K/\partial \alpha_b]$ are neither computed nor stored, rather the V_{bk} are determined directly from the unit local stiffness matrices $[\tilde{k}_{\ell(i)}]$ using the expression

$$V_{bk} = + \sum_{i\epsilon b} \frac{T_{ib(i)}}{\alpha^2_{b(i)}} [\Lambda_i]^T [\tilde{k}_{\ell(i)}] [\Lambda_i] u_{ik} \tag{24}$$

which follows directly from Eqs. (23) and (20). In Eq. (24) it is to be understood that u_{ik} represents the displacement degrees of freedom (in the reference coordinate system) associated with the ith element under the kth load condition.

Using Eq. (24) a set of $B \times K'$ vectors V_{bk} is computed and stored. In general some of these pseudo load vectors will be null or trivially small. In ACCESS 1 the following scheme is employed to delete pseudo load vectors V_{bk} of negligible importance. For load condition k, compute the absolute magnitude $|V_{bk}|$ for $b=1,2,...,B$ and delete V_{bk} from further consideration if

$$|V_{bk}| \le \epsilon' \max_b \{|V_{bk}|\} \tag{25}$$

This deletion process is carried out separately for each load condition $k\epsilon K'$. The result of the deletion process is to reduce the number of V_{bk} vectors from $B \times K'$ to TV_{bk} the number of V_{bk} vectors retained.

Since the decomposed form of the stiffness matrix [see Eq. (21)] is available Eq. (22) may be rewritten as follows

$$[\mathfrak{L}]\ [\mathfrak{D}]\ [\mathfrak{L}]^T \frac{\partial u_k}{\partial \alpha_b} = V_{bk} \tag{26}$$

Now, if the reduced number of V_{bk} vectors, namely TV_{bk}, is less than the number of displacement degrees of freedom in the subset defining the values of the retained constraints (J'), then the partial derivatives desired are obtained by carrying out a sequence of back and forward substitutions for each V_{bk} vector in the reduced set, namely for $(b,k)\epsilon TV_{bk}$. If on the other hand $J' < TV_{bk}$ the reduced set of partial derivatives required is obtained from

$$\frac{\partial u_k}{\partial \alpha_b} = [\tilde{C}]\ V_{bk} \tag{27}$$

where the partial inverse $[\tilde{C}]$ is made up with rows given by C_j^T and the C_j are solutions of

$$[\mathfrak{L}]\ [\mathfrak{D}]\ [\mathfrak{L}]^T C_j = e_j;\quad j\epsilon J' \tag{28}$$

wherein e_j is a vector with all zero elements except for a single unit element corresponding to the jth degree of freedom in the subset J'. It is interesting to note that Eq. (27) can be written in scalar form as follows

$$\frac{\partial u_{\ell k}}{\partial \alpha_b} = \sum_{j=1}^{J} \tilde{C}_{\ell j} V_{jbk};\quad \ell \epsilon J',\ (b,k)\epsilon TV_{bk} \tag{29}$$

which clearly reveals that when the partial inverse branch of the ACCESS 1 program is followed [i.e., when $J' < TV_{bk}$] only the needed partial derivatives are computed and stored.

With the necessary partial derivatives available in storage, it is a straightforward matter to construct explicit approximations for the displacement and stress constraints retained. The stress and displacement constraints are explicit, and in the present instance linear, functions of the response quantities $[u_{jk}, \sigma_{ik}, \sigma_{eik}]$. It is, however, emphasized that the methods used in ACCESS 1 are not restricted to the linear case, rather they only require that the constraints $h_q(\alpha)$ be explicit (continuous and differentiable) functions of the response quantities (see Sec. 2.5.3 of Ref. 16 for a more detailed discussion of this important point).

Explicit approximations of the pertinent displacement response quantities $[\tilde{u}_{jk}(\alpha),\ j\epsilon J',\ k\epsilon K']$ are given by Eq. (17). Explicit approximations for the stress response quantities follow immediately from the displacement approximations [see Eq. (17)], since the pertinent stress-displacement relations are linear and independent of the design variables. For details regarding the explicit approximations for stress response quantities used in ACCESS 1, the reader is referred to Sec. 3.3.6.3 and Appendix A of Ref. 16.

In summary then, the primary formulation of the structural synthesis problem dealt with by ACCESS 1 [see Eqs. (2-7)] is made tractable by: a) reducing the number of design variables via linking leading to the formulation given by Eqs. (12) and (13); b) reducing the number of constraints by temporarily deleting noncritical and redundant constraints; and c) using Taylor series expansions of response quantities to construct explicit approximations for the constraints during any particular stage in the design process. For the pth stage in the design process constraint deletion and construction of explicit approximations converts the basic problem given by Eqs. (12) and (13) to the form, find α such that

$$h_q(\alpha) \approx \tilde{h}_q^{(p)}(\alpha) \ge 0;\quad q\epsilon Q_R^{(p)} \tag{30}$$

and

$$W(\alpha) \to \min \tag{31}$$

where $\tilde{h}_q^{(p)}(\alpha)$ is the explicit approximation of $h_q(\alpha)$ for the pth stage and $Q_R^{(p)}$ denotes the set of constraints to be retained during the pth stage of the design process. Thus the mathematical programing problem faced at each stage in the design process has been rendered small and explicit.

IV. Organization of ACCESS 1

The basic orgainization of the ACCESS 1 computer program is outlined in Fig. 1. The function of the "preprocessor" is to carry out operations that need only be done once at the beginning of a given design problem. These

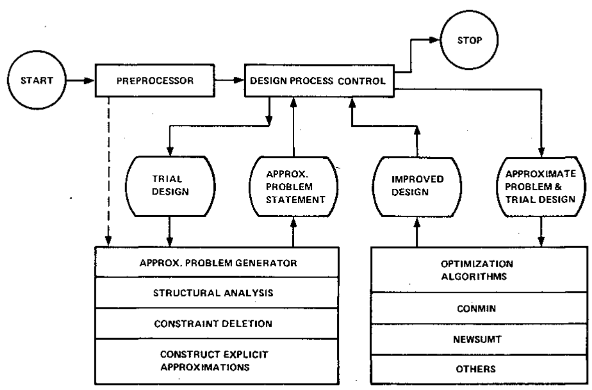

Fig. 1 ACCESS 1 basic organization.

operations are characterized by the fact that they are independent of the design variables.

A typical stage in the ACCESS 1 program begins with the design process control (DPC) block supplying a trial design to the approximate problem generator (APG) block. This trial design is subjected to a detailed finite-element structural analysis for all K load conditions.§ The results of this analysis are then used to evaluate all Q of the constraints in Eq. (12). The number of constraints is then reduced by executing the previously described deletion procedure which temporarily eliminates unnecessary constraints. For the critical and near critical constraints that survive the deletion process, explicit approximations are constructed using Taylor series expansions of the appropriate displacement and stress response quantities in terms of the linked reciprocal design variables α_b. The approximate but explicit representations for the set of constraints retained are passed back to the DPC block which appends the explicit objective function, normalizes the design variable and then gives this approximate, but tractable, mathematical programing problem [see Eqs. (30) and (31)] to the optimization algorithm (OA) block. The design improvement process is now carried out by operating on the current approximate problem statement using either the CONMIN or the NEWSUMPT optimization algorithms. The CONMIN optimizer (see Ref. 20) used in conjunction with a modified Newton method minimizer (see Ref. 21). Either optimizer can be used to generate an improved design or a near optimum solution of the approximate problem supplied by the DPC block for the current stage. It may be noted that in most cases run with the NEWSUMPT optimizer, the tendency has been to set the control parameters so as to seek an improved design, rather than a near optimum solution of the current approximate problem. When this approach is followed is is usually possible to generate a sequence of feasible designs with monotonically decreasing weight because the constraint repulsion characteristic of the interior penalty function tends to generate designs that are not critical with respect to any of the approximated constraints. In any event the last design obtained by the algorithm used is passed back to the DPC block where it becomes the new trial design. This then completes a typical stage in the design process. The iterative multi-stage design process outlined in Fig. 1 is terminated by either a diminishing returns criterion with respect to weight or an overriding termination criterion limiting the total number of stages. Detailed descriptions of the various component parts of the ACCESS 1 program are given in Sec. 3.3 of Ref. 16.

Since the use of approximation concepts has rendered the problems faced at each stage small and explicit, any one of several well-established nonlinear mathematical programing algorithms can be brought to bear on the structural synthesis task. CONMIN (a program for *Con*strained Function *Min*imization) is a general purpose program that has been widely used as a black box optimizer. Relevant literature about the modified feasible directions method implemented by CONMIN will be found in Ref. 19 and further user oriented details are given in Ref. 18. Since the CONMIN package is well documented elsewhere and in view of the fact that all of the results presented herein have been obtained using the NEWSUMT optimization algorithm, it seem appropriate to forego a detailed description of CONMIN.

The NEWSUMT algorithm is a sequence of unconstrained minimization technique based on the extended interior penalty function formulation (see Ref. 20) and the modified Newton method of Ref. 21. The small explicit mathematical programing problem constructed by the approximate problem generator (see Fig. 1), at each stage in the iterative design procedure, has the form given in Eqs. (30) and (31). Using the extended interior penalty function this problem statement is transformed into a sequence of unconstrained minimizations of the following form: Let

$$\phi_e^{(p)}(\alpha,r_a) = W(\alpha) + r_a \sum_{q\in Q_R^{(p)}} \tilde{H}_q^{(p)}(\alpha) \tag{32}$$

where the

$$\tilde{H}_q^{(p)}(\alpha) = \begin{cases} 1/\tilde{h}_q^{(p)}(\alpha) & : \tilde{h}_q^{(p)}(\alpha) \geq \epsilon \\ [2\epsilon - h_q^{(p)}(\alpha)/\epsilon^2] & : \tilde{h}_q^{(p)}(\alpha) < \epsilon \end{cases} \tag{33}$$

and $\phi_e^{(p)}$ is minimized with respect to α for a decreasing sequence of values

$$r_{a+1} = c_a\, r_a; \quad 0 < c_a < 1 \tag{34}$$

In the ACCESS 1 context the functions $\phi_e^{(p)}(\alpha, r_a)$ are defined over the entire positive domain in α space (i.e., for $\alpha_b > 0$; $b = 1,2,\ldots,B$), where they are continuous and have continuous first derivatives. Unconstrained minimization of each function $\phi_e^{(p)}(\alpha, r_a)$ is carried out by executing a series of one-dimensional minimizations according to the prescription

$$\alpha_{m+1} = \alpha_m + d_m^* S_m \tag{35}$$

with the direction vector S_m given by

$$S_m = s_m / |s_m| \tag{36}$$

where

$$s_m = -\left[\frac{\partial^2 \phi_e^{(p)}}{\partial\alpha_b \partial\alpha_c}(\alpha_m, r_a)\right]^{-1} \nabla\phi_e^{(p)}(\alpha_m, r_a) \tag{37}$$

In Eq. (35) the scalar distance d_m^* denotes the solution of the one-dimensional minimization problem. The use of small values for the transition parameter ϵ in ACCESS 1 (see Eq. (33)), leads to large values of $\phi_e^{(p)}(\alpha,r_a)$ when one or more $\tilde{h}_q^{(p)} \leq \epsilon$; $q \in Q_R^{(p)}$. This necessitates the use of robust inherently stable one-dimensional minimization algorithm, namely the golden section method (see for example Ref. 22). It may be of interest to note that an improved method for determining the transition parameter ϵ is presented in Ref. 23.

In ACCESS 1 the transition parameter ϵ in Eq. (33) is always small enough to insure that the solution of each one-dimensional minimization problem resides at a point α_{m+1} where $\tilde{h}_q^{(p)}(\alpha_{m+1}) \geq \epsilon$ for $q \in Q_R^{(p)}$. Therefore, the matrix of second partial derivatives of $\phi_e^{(p)}(\alpha,r_a)$ evaluated at α_m is always given exactly by

$$\left[\frac{\partial^2 \phi_e^{(p)}}{\partial\alpha_b \partial\alpha_c}(\alpha_m, r_a)\right] = \left[\frac{\partial^2 W}{\partial\alpha_b \partial\alpha_c}(\alpha_m)\right] + 2r_a\left[\sum_{q\in Q_R^{(p)}} \frac{\partial\tilde{h}_q}{\partial\alpha_b}(\alpha_m)\frac{\partial\tilde{h}_q}{\partial\alpha_c}(\alpha_m)\right] \tag{38}$$

since in the context of ACCESS 1 the explicit approximations $\tilde{h}_q^{(p)}(\alpha)$ happen to be linear in the linked reciprocal design variables. Because of this linearity the $(\partial^2 \tilde{h}_q^{(p)}/\partial\alpha_b\partial\alpha_c)$ vanish and the first derivatives

$$\frac{\partial\tilde{h}_q}{\partial\alpha_b}(\alpha), \; \frac{\partial\tilde{h}_q}{\partial\alpha_c}(\alpha)$$

are invariant for the pth approximate problem statement. It is also observed that for α_m in the positive domain (see Eq. (16)) the matrix

$$\left[\frac{\partial^2 W}{\partial\alpha_b \partial\alpha_c}(\alpha_m)\right]$$

§If the trial design is unacceptable with respect to any of the constraints, it is scaled up uniformly to satisfy the most critical constraint with a small margin.

in Eq. (38) is always diagonal and positve definite which tends to improve the conditioning of

$$\left[\frac{\partial^2 \phi_e}{\partial \alpha_b \partial \alpha_c} (\alpha_m, r_a) \right]$$

The optimization program NEWSUMT was written to implement the modified Newton method as an ACCESS 1 program option. Initially, the scalar factor r_a is given a value such that ϕ_e is equal to twice the value of the weight W [see Eq. (32)]. The scalar r_a is decreased by a prespecified ratio (c_a) every time minimization for the current value r_a is completed. The approximate problem statement is updated after a prespecified number of unconstrained minimizations (a'). It is important to emphasize that ACCESS 1 permits the user to specify the cut factor (c_a) and the number of unconstrained minimizations (a') to be executed prior to updating the approximate problem statement. Representative numbers for these control parameters are $c_a = 0.3$ and $a' = 2$.

V. Sample Applications of ACCESS 1

In this section results for three example problems will be presented in brief summary form. The examples shown here are drawn from an extensive collection of detailed results presented in Ref. 16. Herein emphasis is placed on illustrating the efficiency of the new analysis/synthesis capability represented by the ACCESS 1 computer program. Therefore, material distribution and critical constraint data for the final designs obtained are omitted due to space limitations. Those interested in the complete problem statements and detailed results are referred to Ref. 16.

Wing Carry Through Truss

The first example problem was set forth in Ref. 24 and it involves a 63 bar truss-element idealization of the titanium alloy wing carry through box for a heavy swing wing aircraft subject to two distinct load conditions (see Fig. 2 where, for the sake of clarity some members are not shown). The material properties used are $\rho = 0.16$ lb/in.^{3}, $E = 16 \times 10^6$ lb/in.^{2} and $\sigma_{ai} = \pm 100{,}000$ lb/in.^{2}. Since design variable linking is eschewed the problem involves 63 independent design variables, one A_i for each truss member. The structure is supported at nodes 15, 16, 17, and 18 and the displacement method analysis involves 42 degrees of freedom. The initial size and minimum area of all members are 20 in.^{2} and 0.01 in.^{2}, respectively. Initially, minimum weight designs are sought considering only stress and minimum size constraints (Problem 1A), using two different control parameter settings, namely $c_a = 0.5$ with $a' = 2$ and $c_a = 0.05$ with $a' = 1$. Note that in Table 1 these control parameter settings are abbreviated to (0.5×2) and (0.05×1), respectively. Subsequently, minimum weight designs are sought considering the same stress and minimum size constraints but adding a torsional rotation constraint (Problem 1B) by requiring that the relative displacement of nodes 1 and 2 in the x direction be less than 1.0 in. Note that a minor modification of the standard ACCESS 1 program was required to handle this torsional rotation constraint. Results for Problem 1B have been obtained (see Table 1 and Fig. 3) using the same control parameter settings, namely (0.5×2) and (0.05×1). The final material distributions obtained for Problems 1A and 1B using ACCESS 1-NEWSUMT (with two different control

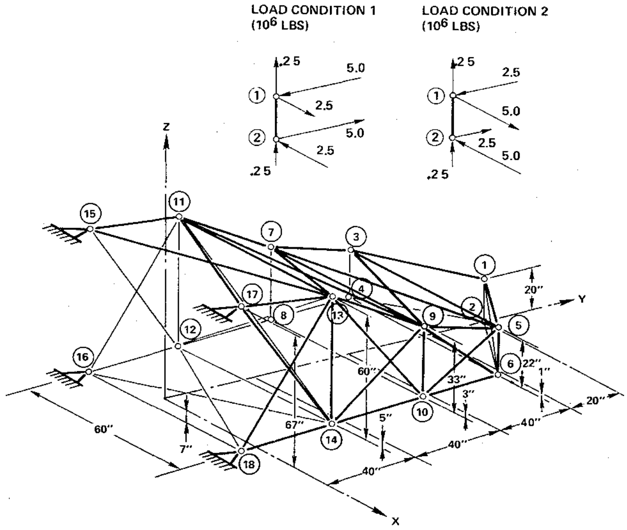

Fig. 2 Truss idealization of wing carry through structure.

Table 1 Summary of ACCESS 1 results

	Problem 1 Wing carry-through truss				Problem 2 Swept wing				Problem 3 Delta wing				
	1A Stress		1B Stress & displ.		2A Initial design		2B Initial design		3A	3B	3C Initial design	3D	3E
	.5×2	0.05×1	.5×2	0.05×1	I	II	I	II	II	I	I	I	I
No. of elements			63		130	130	150	150			133		
No. of D.O.F.'s[a]			42				120				105		
No. of L.C.'s			2				2				1		
No. of D.V.'s			63		18	18	32	32	28	22	28	40	56
Initial weight (lb)			11104.7		3277	4959	3282	4964	21193	15800	15800	15800	15800
Weight after 5 analyses (lb)	5556	5065	6593	6182	2469	2469	2468	2468	10775	10843	10764	9851	9746
Final weight (lb)	4977	5008	6119	6153	2464	2463	2463	2461	10742	10824	10741	9636	9521
Total number of analyses	15	10	15	10	8	8	8	8	9	9	9	11	12
Time in APG[b] (sec)	22.1	16.1	21.5	13.9	15.0	17.0	23.6	22.9	11.6	10.5	11.5	15.8	24.7
Time in[b] optimizer (sec)	69.1	35.2	75.5	30.3	2.7	3.1	10.8	10.2	5.4	3.3	5.4	15.4	41.5
Total time[b] (sec)	92.0	52.1	98.1	45.0	19.1	21.5	36.0	34.6	17.9	14.7	17.8	32.3	67.8

[a] Free displacement degrees of freedom.
[b] CPU time on IBM 360/91 at UCLA, FORTRAN-H.

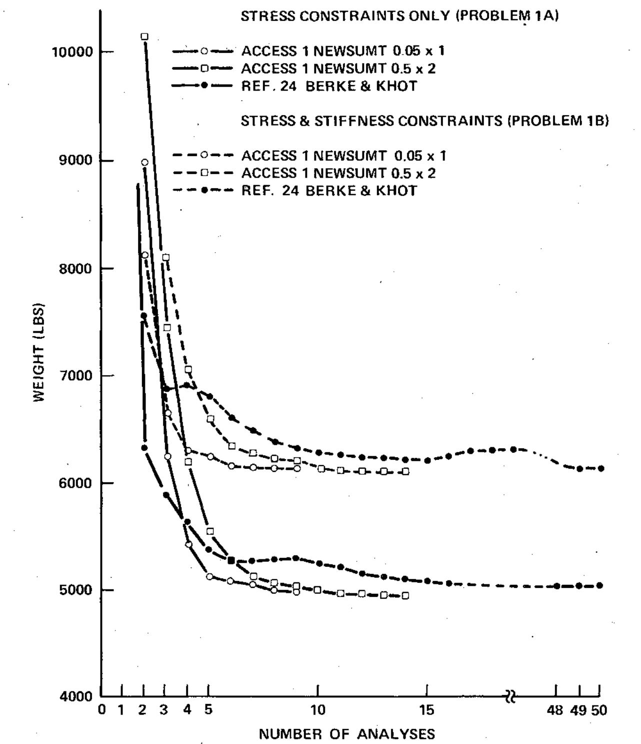

Fig. 3 Iteration histories for problem 1 wing carry through truss (from Ref. 24).

parameter settings (0.5 × 2) and (0.05 × 1)) are very similar to one another and they compared well with the material distributions reported in Ref. 24. Detailed material distribution data and numerical iteration history data are given in Tables 46 and 47 of Ref. 16. The iteration histories plotted in Fig. 3 show that, for the 63 bar wing carry through truss example problem, ACCESS 1 is competitive with the automated redesign methods applied to this problem in Ref. 24.

Swept Wing Example

The second example is an idealized representation of a 10% thickness ratio aluminum alloy swept wing (see Fig. 4) subject to two distinct loading conditions. The structure is taken to be symmetric with respect to the x-y plane which corresponds to the wing middle surface. The upper half of the wing is initially modeled using 60 CST elements to represent the skin and 70 SSP elements for the vertical webs. Subsequently, 20 TRUSS elements are added to represent forward and aft spar caps (see cross-sectional view in Fig. 4). Extensive but plausible design variable linking is employed and the number of independent design variables describing the skin, web, and spar cap material distributions are 7, 11, and 14, respectively. Therefore, when the wing is modeled using only CST and SSP elements the problem involves a total of 18 design variables and it is herein designated as Problem 2A. When the 20 truss elements representing the two spar caps are added, the total number of independent design variables increases to 32 and this problem is herein designated as Problem 2B. The wing is subject to two distinct static load conditions. The first load condition is roughly equivalent to a uniform pressure loading of 80 lb/ft^2 and the second involves the same total loading with the distribution changed so that the center of pressure is moved forward. The material properties used are $\rho = 0.096$ lb/in.^{3}, $E = 10 \times 10^6$ lb/in.^{2}, $\upsilon = 0.3$ and $\sigma_{ai} = \pm 25{,}000$ lb/in.^{2}. The wing tip deflection at nodes 41, 42, 43, and 44 (see Fig. 4) is constrained to be less than 60 in. Both lower ($A_i^{(L)} = 0.01$ in.^{2}) and upper ($A_i^{(U)} = 1.50$ in.^{2}) limits are imposed on the TRUSS member sizes. For CST and SSP elements only minimum thickness limits are specified (i.e., $t_i^{(L)} = 0.02$ in.). the idealized swept wing depicted in Fig. 4 is supported at the root by setting all displacement components at nodes 1, 2, 3, and 4 to zero. Referring to Fig. 4 it is apparent that, independent of whether or not spar caps are included, the number of displacement degrees of freedom involved in the structural analysis is 120. Without spar caps (Problem 2A) the problem involves 130 finite elements and with spar caps (Problem 2B) the problem involves 150 finite elements.

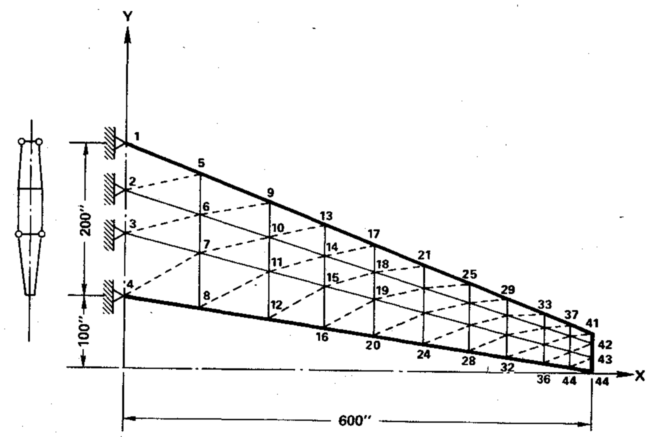

Fig. 4 Swept wing example (problem 2).

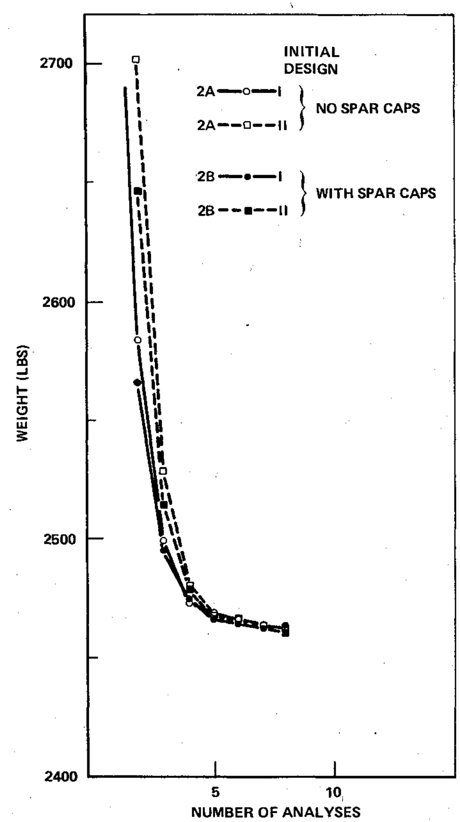

Fig. 5 Iteration histories for problem 2 swept wing.

Results for Problems 2A and 2B have been obtained from two distinct starting point designs (see Table 1). Iteration histories for the ACCESS 1 solutions of Problems 2A and 2B from starting point designs I and II are shown in Fig. 5. In both problems it is found that the skin material distributions are for practical purposes the same independent of the starting design used. Furthermore, the skin material distributions found for Problems 2A and 2B are substantially the same, except for the fact that the panel nearest the root is approximately 0.9% thicker when the spar caps are omitted

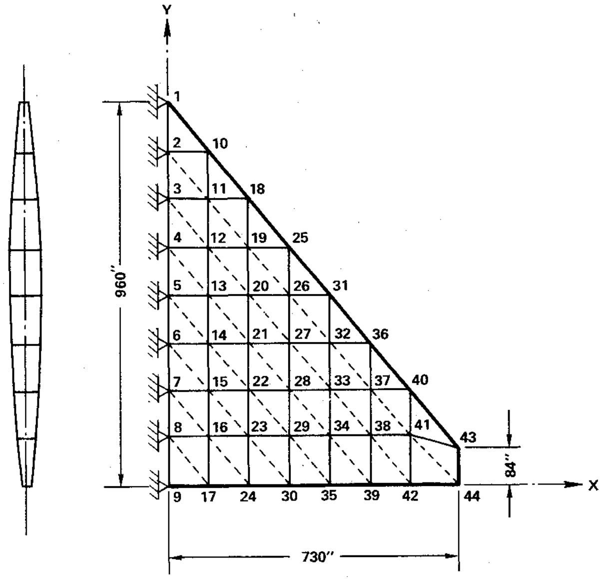

Fig. 6 Delta wing example (problem 3).

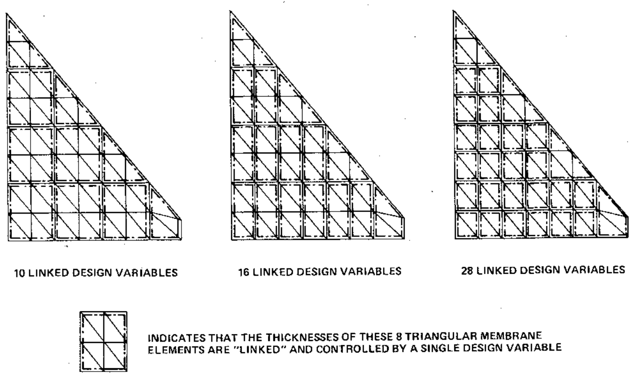

Fig. 7 Alternate linking arrangements (skin).

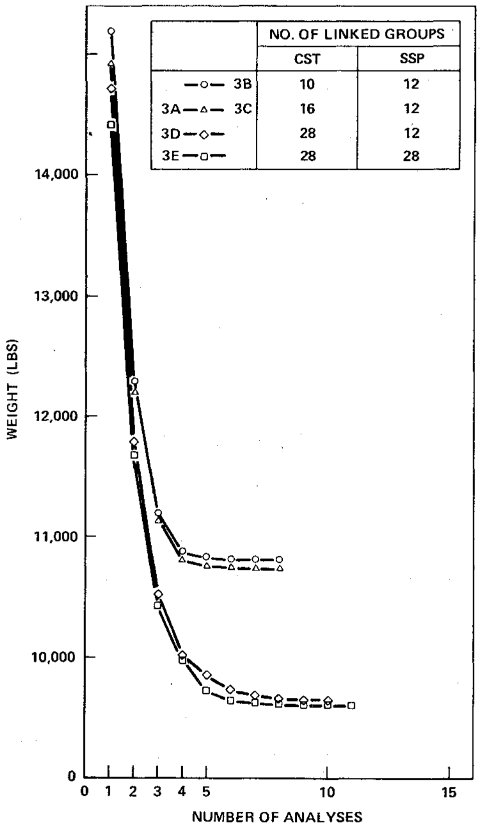

Fig. 8 Iteration histories for problem 3 delta wing.

(i.e., in Problem 2A). The vertical shear web and spar cap material distributions exhibit some local differences when results obtained from two different starting points are compared. However, the shear web and spar cap material, respectively, account for less than 12.5 and 2.5 percent of the final weight. Consequently, the final wing weights achieved without spar caps (Problem 2A) differ by only 0.06% while the final wing weights obtained with spar caps differ by only 0.09%. It is interesting to note that in this example adding 20 TRUSS elements, to represent forward and aft spar caps, does not lead to a significant weight reduction.

Examination of Fig. 5 reveals that the swept wing design iteration histories converge rapidly. For the four cases shown in Fig. 5, a feasible design that is within 1% of the minimum weight achieved, was obtained by ACCESS 1 after no more than 5 analyses. Detailed material distribution and numerical iteration history data for these swept wing examples are given in Tables 58 and 59 of Ref. 16.

Delta Wing Example

The third example is an idealized thin (3% thickness ratio) titanium delta wing structure (see Fig. 6) similar to that considered in Refs. 21 and 25. The structure is assumed to be symmetric with respect to its middle surface which corresponds to the *x-y* plane. The upper half of this wing is modeled using 63 CST elements to represent the skin and 70 SSP elements for the vertical webs. The wing is subject to a single static load condition that is roughly equivalent to a uniformly distributed loading of 144 lb/ft^2. The material properties used are $\rho = 0.16$ lb/in.^{3}, $E = 16.4 \times 10^6$ lb/in.^{2},- $\nu = 0.3$, and $\sigma_{ai} = \pm 125{,}000$ lb/in.^{2} Minimum gauge limits of 0.02 in. are specified for all CST and SSP elements. Deflection constraints are imposed on the transverse displacements of the wing at all free nodes (i.e., 10-44). These displacement constraints form a linear deflection constraint envelope varying from 100.8 in. at the wing tip to zero at the root. Note that if only tip displacements of nodes near the leading and trailing edges could become excessive (equal or exceed the tip deflection).

The 133 finite-element representation of the delta wing shown in Fig. 6 is supported by fixing nodes 1 though 9 and it is apparent that the displacement method structural analyses will involve 105 displacement degrees of freedom. Various design variable linking models are employed. In the skin, three distinct linking arrangements leading to 10, 16, and 28 independent design variables, respectively, are employed (see Fig. 7). For the webs two alternative linking arrangements involving 12 and 28 independent design variables, respectively, are employed.

Results for five distinct problems (3A thorugh 3E) are summarized in Table 1 and the corresponding interation histories are shown in Fig. 8. Problems 3A and 3C involve 28 independent design variables, 16 for the skin (CST's) and 12 for the webs (SSP's), and they differ only with respect to the starting points used (3A initial design I all CST's 0.10 in., all SSP's 0.15 in.; 3C initial design II all CST's 0.15 in., all SSP's 0.12 in.). Problems 3B, and 3D, and 3E, respectively, involve 22, 40, and 56 independent design variables after linking with 10, 28, and 28 for the skin (CST's) and 12, 12, and 28 for the webs (SSP's). These three problems (3B, 3D, and 3E) are started from initial design I.

The iteration histories for problems 3A and 3C are so much alike that they appear as a single line in Fig. 8, even though different initial designs were used. Comparing the final designs obtained for Problems 3A and 3C it is found that both the skin and the web material distributions are essentially identical to three decimal places and the final design weights achieved are the same to four significant figures. Examination of the results for the remaining cases (3B, 3D, and 3E) shows

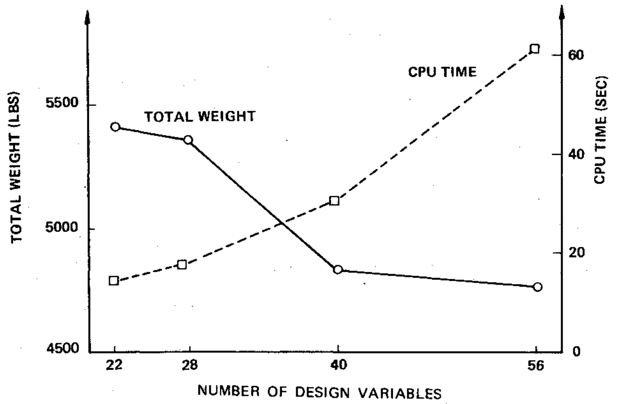

Fig. 9 Minimum weight achieved and CPU time required versus number of design variables for problem 3.

that the achievable minimum weight decreases as the number of design variables is increased while the CPU time required to converge increases. This is illustrated graphically in Fig. 9. Comparing the final weights achieved in Problems 3C and 3D it is observed that increasing the number of design variables in the skin from 16 to 28 leads to a 10% weight reduction. This is not surprising since for each of the five Problems 3A through 3E the skin accounts for approximately 92% of the total weight at the final design.

Figure 8 shows that the delta wing design iteration histories converge rapidly. For Problems 3A, 3B and 3C feasible designs within 1% of the corresponding minimum weight achieved, are obtained after only 4 analyses. In Problems 3D and 3E feasible designs within 1% of the corresponding minimum weight achieved are obtained after 6 and 7 analyses, respectively. These delta wing results further illustrate the efficiency of the ACCESS 1 computer program. Detailed material distribution and numerical iteration history data for Problems 3A through 3E are given in Tables 67 and 68 of Ref. 16.

Additional Data on Examples

A summary of the results presented, that includes some CPU run time data, is given in Table 1. The CPU time data is presented for completeness, however, comparisons based on CPU time and/or run time costs are avoided. This is because such comparisons can be misleading unless the alternative programs are run on the same installation at nearly the same time (assuming a time shared operating environment). All of the example problems reported here were executed on the IBM 360/91 at the UCLA Campus Computing Network, using an object program compiled by the FORTRAN H compiler. Total CPU times as well as the CPU times spent in the two major parts of the design process, approximate problem generation (APG) and optimization algorithm (OA), are given in Table 1. Examining Table 1 it is first observed that the total run times are modest in relation to problem size. Furthermore, the distribution of effort between the approximate problem generator (APG) and the optimization algorithm (OA) portions of the ACCESS 1 program (see Fig. 1) is reasonably well balanced. It is emphasized that the APG block in Fig. 1 includes structural analysis, constraint deletion, and generation of explicit approximations.

It should be kept in mind that elapsed CPU time data tends to be rather sensitive to control parameters (e.g., c_a and a') and convergence criteria. This is illustrated in Fig. 10 by plotting total weight versus elapsed CPU time for problem 3C with $c_a=0.05$ and $a'=1$ rather than the conservative values previously used (i.e., $c_a=0.3$ and $a'=2$). Modifying the control parameters is seen to reduce the reportable run time from 17.83 sec. to 14.34 sec. and the final design obtained is only 10.1 lb heavier. The preassigned criterion that terminated both of the runs plotted in Fig. 10 is that the percentage

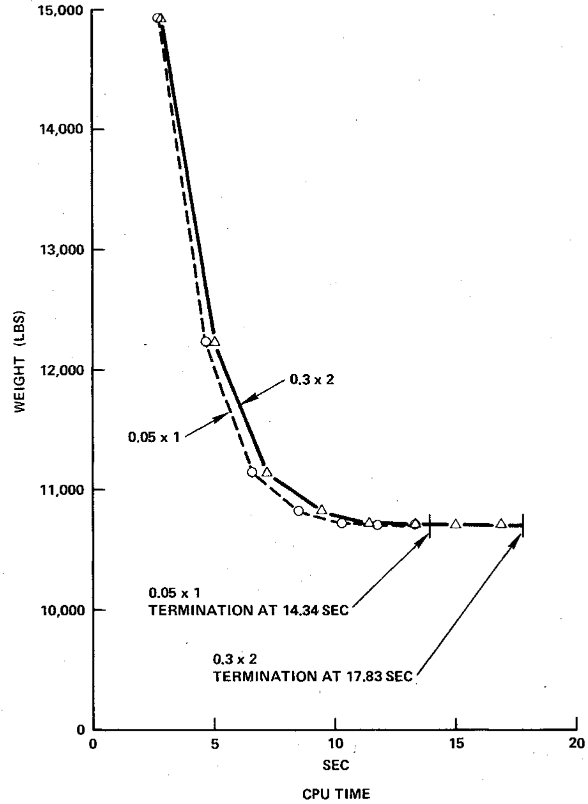

Fig. 10 Weight vs elapsed CPU time delta wing problem 3c.

change in the objective function value for two successive stages was less than 0.1%. If for the run with $c_a=0.05$ and $a'=1$, the termination criterion was taken to be, stop when the reduction in the objective function value achieved in a single stage is less than 1%, then convergence would occur after only 5 stages. The reportable run time would then be 10.35 sec. and the final weight obtained would be only 19.3 lb heavier than that achieved after 17.83 sec using the control parameters $c_a=0.3$ and $a'=2$ with the previously defined tighter convergence criterion.

VI. Conclusions

It has been demonstrated that efficient structural synthesis capabilities based on combining finite-element structural analysis methods and nonlinear mathematical programing techniques can be generated. The coordinated implementation of various approximation concepts has made it possible to achieve excellent efficiency while retaining the philosophically attractive generality inherent to the mathematical programing formulation of structural design optimization problems.

Based on the ACCESS 1 numerical results presented here and in Ref. 16 two major conclusions are drawn:

1) The innovative use of approximation concepts has produced a dramatic reduction in the number of conventional structural analyses needed to obtain candidate optimum designs via the combined use of finite element and mathematical programing methods. Indeed, the numerical results reported herein indicate that ACCESS 1 is usually able to obtain a practical near optimum design within 5 to 10 analyses.

2) For structural synthesis problems of modest but useful size, approximation concepts have made possible the generation of an automated structural design capability, based on finite-element analysis mathematical programing algorithms, that is competitive with recursive redesign techniques based on fully stressed design and discretized optimality criteria concepts (particularly when measured by the number of analyses required to obtain an optimum design). It

should be noted that as analysis problem size grows (more elements, degrees of freedom and load conditions), the argument for using the number of analyses to convergence as a measure of optimization efficiency is strengthened.

The central notion of replacing the mathematical programing statement of the design optimization problem with a sequence of small explicit approximate problems (see Fig. 1), that retain the essential features of the primary problem, should be widely applicable in structural synthesis.

The basic ideas used in creating the ACCESS 1 program are rather general and therefore it may be argued that its successful development supports the contention that the introduction of approximation concepts will lead to the emergence of a new generation of practical and efficient large scale structural synthesis capabilities based on finite-element analysis and mathematical programing algorithms.

References

[1]Schmit, L. A., "Structural Design by Systematic Synthesis," *Proceedings of the 2nd Conference on Electronic Computation ASCE*, New York, 1960, pp. 105-122.

[2]Gellatly, R. A., "Development of Procedures for Large Scale Automated Minimum Weight Structural Design," Air Force Flight Dynamics Lab., Wright-Patterson AFB, Ohio, AFFDL-TR-66-180, 1966.

[3]Karnes, R. N. and Tocher, J. L., "Automatic Design of Optimum Hole Reinforcement," Rept. D6-23359, The Boeing Company, Commercial Airplane Division, Renton, Washington, 1968.

[4]Tocher, J. L. and Karnes, R. N., "The Impact of Automated Structural Optimization on Actual Design," AIAA Paper 71-361, Apr. 1971, Anaheim, Calif.

[5]Venkayya, V. B., Khot, N. S., and Reddy, V. S., "Optimization of Structures Based on the Study of Strain Energy Distribution," *Proceedings of the 2nd Conference on Matrix Methods in Structural Mechanics*, Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio, AFFDL-TR-68-150, Dec. 1969, pp. 111-153.

[6]Dwyer, W., Rosenbaum, J., Shulman, M., and Pardo, H., "Fully Stressed Design of Airframe Redundant Structures," *Proceedings of the 2nd Conference on Matrix Methods in Structural Mechanics*, Wright Patterson Air Force Base, Ohio, AFFDL-TR-68-150, Dec. 1969, pp. 155-181.

[7]Lansing, W., Dwyer, W., Emerton, R., and Ranalli, E., "Application of Fully-Stressed Design Procedures to Wing and Empennage Structures," *Journal of Aircraft*, Vol. 8, Sept. 1971, pp. 683-688.

[8]Venkayya, V. B., "Design of Optimum Structures," *Journal of Computers and Structures*, Vol. 1, No. 1-2, Aug. 1971, pp. 265-309.

[9]Gallatly, R. A., Berke, L., and Gibson, W., "The Use of Optimality Criteria in Automated Structural Design," Paper presented at the 3rd Conference on Matrix Methods in Structural Mechanics, Wright Patterson AFB, Ohio, Oct. 1971.

[10]Dwyer, W. J., Emerton, R., and Ojalvo, I. U., "An Automated Procedure for the Optimization of Practical Aerospace Structures," Air Force Flight Dynamics Lab., Wright Patterson AFB, Ohio, AFFDL-TR-70-118, Mar. 1971.

[11]Gellatly, R.A. and Berke, L., "Optimal Structural Design," Air Force Flight Dynamics Lab., Wright Patterson AFB, Ohio, AFFDL-TR-70-165, 1971.

[12]Taig, I. D. and Kerr, R. I., "Optimization of Aircraft Structures with Multiple Stiffness Requirements," *AGARD Conference Proceedings No. 123, 2nd Symposium on Structural Optimization*, Milan, Italy, Apr. 1973.

[13]Venkayya, V. B., Khot, N. S., and Berke, L., "Application of Optimality Criteria Approaches to Automated Design of Large Practical Structural Optimization," Milan, Italy, Apr. 1973.

[14]Schmit, L. A. and Farshi, B., "Some Approximation Concepts for Structural Synthesis," *AIAA Journal*, Vol. 12, May 1974, pp. 692-699.

[15]Baldur, R., "Structural Optimization by Inscribed Hyperspheres," *Journal of the Engineering Mechanics Division, ASCE*, Vol. 98, No. EM3, 1972, pp. 503-518.

[16]Schmit, L. A. and Miura, H., "Approximation Concepts for Efficient Structural Synthesis," NASA CR-2552, Mar. 1976.

[17]Melosh, R. J. and Bamford, R. M., "Efficient Solution of Load Deflection Equations," *Journal of The Structural Division, ASCE*, Vol. 95, No. ST4, Apr. 1969, pp. 661-675.

[18]Vanderplaats, G.N., "CONMIN-A FORTRAN Program for constrained Function Minimization-User's Manual," NASA TMX-62-282, Aug. 1973.

[19]Vanderplaats, G. N. and Moses, F., "Structural Optimization by Methods of Feasible Directions," *Journal of Computers and Structures*, Vol. 3, No. 4, July 1973, pp. 739-755.

[20]Kavlie, D. and Moe, J., "Automated Design of Frame Structures," *Journal of the Structural Div.*, ASCE, Vol. 97, No. ST1, 1971, pp. 33-62.

[21]Haftka, R., "Automated Procedure for Design of Wing Structures to Satisfy Strength and Flutter Requirements," NASA TN D-7264, July 1973.

[22]Wilde, D. J., *Optimum Seeking Methods*, Prentice Hall, Englewood Cliffs, N.J., 1964.

[23]Cassis, J. H. and Schmit, L. A., "On Implementation of the Extended Interior Penalty Function," *International Journal for Numerical Methods in Engineering*, Vol. 10, No. 1, 1976, pp. 3-23.

[24]Berke, L. and Khot, N. S., "Use of Optimality Criteria Methods for Large Scale Systems," AGARD Lecture Series No. 70 on Structural Optimization, AGARD-LS-70, 1974, pp. 1-29.

[25]Stroud, W. J., Dexter, C. G., and Stein, M., "Automated Preliminary Design of Simplified Wing Structures to Satisfy Strength and Flutter Requirements," NASA TN D-6534, Dec. 1971.